Experimental Demonstration of Time-Delay Interferometry for the Laser Interferometer Space Antenna

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We report on the first demonstration of time-delay interferometry (TDI) for LISA, the Laser Interferometer Space Antenna. TDI was implemented in a laboratory experiment designed to mimic the noise couplings that will occur in LISA. TDI suppressed laser frequency noise by approximately 10^9 and clock phase noise by 6×10^4, recovering the intrinsic displacement noise floor of our laboratory test bed. This removal of laser frequency noise and clock phase noise in postprocessing marks the first experimental validation of the LISA measurement scheme.

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The Laser Interferometer Space Antenna (LISA) [1] is a joint National Aeronautics and Space Administration (NASA) and European Space Agency (ESA) gravitational wave observatory. LISA will observe gravitational radiation from massive black hole mergers out to a distance of $z = 20$; black hole, neutron star, and white-dwarf inspirals into massive black holes; and white-dwarf binary orbits throughout the galaxy [2].

LISA will measure the relative motion of three drag-free spacecraft (SC) separated by 5×10^9 m with a one-way resolution of $2 \times 10^{-11}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ ($4 \times 10^{-21}/\sqrt{\mathrm{Hz}}$ strain sensitivity). Laser light is passed between SC and the interference phase between the local and distant laser (one-way) phases recorded. The design sensitivity is dominated by shot noise from the laser light for frequencies above 3 mHz and by spurious forces on the proof masses at lower frequencies.

Orbital motion of the SC Doppler shifts the laser beams by up to 20 MHz, giving rise to heterodyne signals upon interference with a local oscillator on each SC. The phase change of these beat note signals is proportional to the change in path length between SC. Gravitational waves also cause a displacement between SC, phase shifting the beat note. The challenge for LISA is to measure these phase shifts with μcycle accuracy in the presence of slowly varying Doppler shifts, millions of cycles of laser frequency noise and variations in the clock sampling frequencies.

The LISA arm lengths will neither be matched (the mismatch will be up to 75 000 km) nor static, introducing sensitivity to laser frequency noise. LISA will use a technique called time-delay interferometry (TDI), combining local and interspacecraft phase measurements in postprocessing, to form configurations equivalent to Michelson and Sagnac interferometers [3]. TDI suppresses noise from laser frequency fluctuations by many orders of magnitude, yet preserves the gravitational wave signal. TDI consists of linear combinations of the phase measurements recorded at specific times determined by the light travel time between SC. TDI will also correct for phase noise of the ultrastable oscillators (or clocks), which provide the phase measurement references on each SC.

Although TDI has been extensively studied theoretically, there have not previously been any experimental demonstrations of the key aspects of the signal processing chain. This Letter reports results from the first demonstration of TDI in a laboratory experiment designed to mimic the noise couplings that will occur in LISA. This experiment was set up to resemble two LISA SC, each with two lasers and a phase meter referenced to an independent clock. The results show that TDI suppressed laser frequency fluctuations by 10^9 at 3 mHz and clock noise by 6×10^4. This confirms the LISA measurement scheme and validates the performance of the LISA phase meter [4] with a LISA-like signal structure.

TDI can be understood as a technique to synthesize equal arm-length interferometer configurations from one-way measurements. With equal lengths, the effect of laser frequency noise is common to each arm of a two beam interferometer and will cancel when differenced.

To illustrate the concept of a synthetic configuration, we show how a round-trip measurement, Fig. 1(a), can be made by combining two one-way measurements, Fig. 1(b). In Fig. 1(a) a laser beam travels a distance L_{12} from SC1 to SC2, where it is retroreflected, traveling a further distance, L_{21}. The phase of the return beam is measured relative to the outgoing beam, $\phi_r(t) = D_{21}D_{12}p_{21}(t) - p_{21}(t)$, where $p_{ij}(t)$ is the output laser phase on SC j looking at SC i. D_{ij} is a delay operator [5] representing the application of a time delay: $D_{ij}a(t) = a(t - L_{ij}/c)$. In this Letter we deal only with static arm lengths; therefore delay operators commute, $[D_i, D_j] = 0$. For clarity, noise sources other than laser phase noise are neglected, until Eq. (2). The same information in $\phi_r(t)$ can be acquired by making two one-way measurements [Fig. 1(b)] and combining them with a time delay determined by the light travel time. LISA employs this symmetric arrangement, with a laser and a phase measurement made at each end. The measurements at SC1 and 2, re-

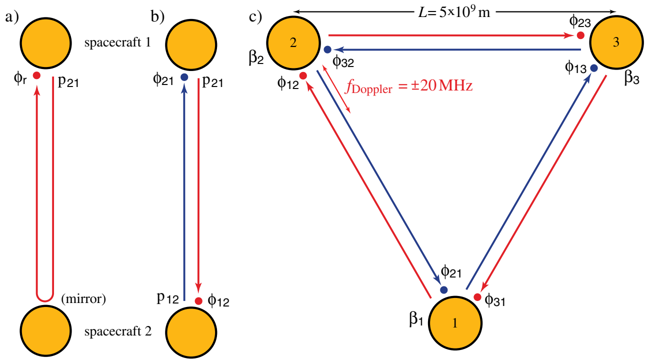

FIG. 1 (color online). Schematic representation of laser links between spacecraft. (a) single round-trip and (b) one-way phase measurements. (c) LISA constellation showing the six (one-way) interspacecraft phase measurements. All spacecraft are identical.

spectively, are $\phi_{21}(t) = D_{21}p_{12}(t) - p_{21}(t)$ and $\phi_{12}(t) = D_{12}p_{21}(t) - p_{12}(t)$ [6]. Combining these one-way links with a time shift gives the same result as the conventional mirror-based measurement: $\phi_{21}(t) + D_{21}\phi_{12}(t) = \phi_r(t)$.

Extending this approach to multiple links allows numerous configurations to be formed, including combinations that maintain equal arm lengths in the presence of SC velocity [7,8]. A conventional Sagnac interferometer would be formed by interfering two laser beams that have counterpropagated around the constellation. The same information is contained in the TDI combination, α [9]:

$$\begin{aligned}\alpha(t) = {} & \phi_{31}(t) + D_{31}(\phi_{23}(t) - \beta_3(t)) + D_{23}D_{31}(\phi_{12}(t) \\ & - \beta_2(t)) - \phi_{21}(t) - D_{21}(\phi_{32}(t) + \beta_2(t)) \\ & - D_{32}D_{21}(\phi_{13}(t) + \beta_3(t)) \\ & - (1 + D_{12}D_{23}D_{31})\beta_1(t), \end{aligned} \tag{1}$$

In Eq. (1) we have included the back-link measurements $\beta_j(t)$ made on SC j, a measure of the phase difference between the two local lasers: $\beta_1(t) = p_{21}(t) - p_{31}(t)$, $\beta_2(t) = p_{32}(t) - p_{12}(t)$, and $\beta_3(t) = p_{13}(t) - p_{23}(t)$. The phases of the interspacecraft links from SC i to SC j are $\phi_{ij}(t) = D_{ij}p_{ji}(t) - p_{ij}(t)$.

The phase measurements will be triggered by an onboard clock. As such the phase measurements taken on separate SC will not be taken simultaneously [this effect is not included in Eq. (1)]. To adequately suppress the frequency noise, described by Eq. (1), in addition to correcting for the light travel time between spacecraft, the delay operators must correct for the relative clock offset, to the level of ~3 ns [10]. LISA will use a dedicated system to determine both the light travel times and the clock offsets. The phase measurements will be sampled at ≈3 Hz and later shifted to the required times using ns-accuracy interpolation algorithms [11].

The spacecraft clocks also introduce clock phase noise into each phase measurement. Clock phase fluctuations are indistinguishable from beat note fluctuations, which contain the gravitational wave signal. To measure the phase of a 20 MHz (maximum Doppler shift, f_0) beat note with a phase sensitivity, $\phi(f) = 10^{-6}$ cycles$/\sqrt{\text{Hz}}$ at $f = 3$ mHz, LISA requires a clock stability of $y(f) = 2\pi f \phi(f)/f_0 = 9 \times 10^{-16}/\sqrt{\text{Hz}}$.

With the required stability unavailable for spaceflight, clock noise will be measured and removed, by transferring the clock phase between spacecraft with a fidelity of μ cycles$/\sqrt{\text{Hz}}$ [12,13]. To improve the signal to noise ratio, the clock signal will be multiplied to a frequency of several GHz and phase modulated onto the laser. Interference between the distant and local lasers' sidebands appears in the photodetector signal near the carrier-carrier beat note frequency. These sideband-sideband beat notes give a measure of the clock noise, which can be removed by modified TDI combinations in processing.

The LISA interferometry test bed (Fig. 2), based on the TDI Sagnac combination α, is designed to provide signals representative of LISA interferometry, without requiring 5×10^6 km arms. The experiment reproduces the essential experimental complexity of LISA: polarization leakage interferometry; multiple heterodyne frequencies; independent clocks; and independent phase measurements of optical signals and clock sidebands. Multiple heterodyne frequencies are required to avoid artificial common-mode cancellation of nonlinear effects. A representative signal structure for testing these concepts is obtained with two optical benches.

With only two benches in our experiment, the α combination must be modified. This is equivalent to the lasers on the third SC being phase locked according to the scheme presented in [9]. In this case the interspacecraft links previously containing SC3 become $\phi_{32}(t) = D_{32}D_{13}p_{31}(t) - p_{32}(t)$, $\phi_{31}(t) = D_{23}D_{31}p_{32}(t) - p_{31}(t)$,

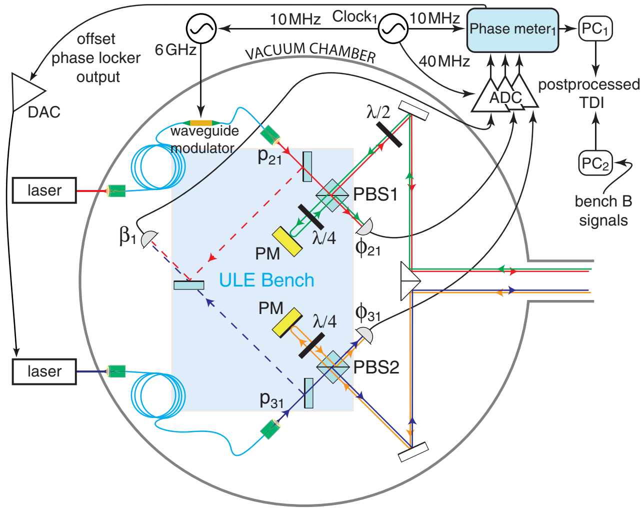

FIG. 2 (color online). Schematic of one bench of the LISA interferometry test bed (the other bench is an identical copy). The benches are mechanically coupled via a vacuum envelope. $\lambda/4$ = quarter wave plate, $\lambda/2$ = half wave plate, PBS = polarizing beam splitter, PM = proof mass, DAC = digital-to-analog converter, ADC = analog-to-digital converter.

with $\beta_3(t) = \phi_{23}(t) = \phi_{13}(t) = 0$. Assuming two SC with independent clocks, Eq. (1) becomes

$$\begin{aligned}\alpha^c(t_1, t_2) = {} & \phi^c_{31}(t_1) + D_{23}D_{31}(\phi^c_{12}(t_2) - \beta^c_2(t_2)) - \phi^c_{21}(t_1)\\ & - D_{21}(\phi^c_{32}(t_2) + \beta^c_2(t_2))\\ & - (1 + D_{12}D_{23}D_{31})\beta^c_1(t_1),\end{aligned} \tag{2}$$

where the notation change $\phi_{ij}(t) \rightarrow \phi^c_{ij}(t_j)$ indicates that each measurement is made at time t_j, triggered by the clock on SC j, and the superscript c denotes a phase measurement containing clock phase noise. The normalized clock time error $q_j(t_j)$ enters each phase measurement made on SC j in direct proportion to its heterodyne beat note frequency. The single link and back-link measurements become, for example,

$$\begin{aligned}\phi^c_{ij}(t_j) &= \phi_{ij}(t_j) - q_j(t_j)(\nu_{ji}(t_j) - \nu_{ij}(t_j))\\ \beta^c_j(t_j) &= \beta_j(t_j) - q_j(t_j)(\nu_{ij}(t_j) - \nu_{kj}(t_j)),\end{aligned} \tag{3}$$

where $\nu_{ij}(t)$ is the optical frequency of the ij laser.

The experiment comprises two simplified LISA-topology optical benches constructed from ultralow expansion (ULE) glass (170 × 120 × 20 mm), separated by 1.0 m, and occupying a common vacuum envelope ($\sim$1 mTorr). For sensitive optical paths, fused silica components were optically contacted onto the ULE bench. This technique has shown picometer-level displacement stability on a single bench [14]. Each optical bench represents a single spacecraft; one is illustrated in Fig. 2. The two benches each have two lasers and three phase measurements: two interbench, ϕ_{21} and ϕ_{31} on bench 1, and ϕ_{12} and ϕ_{32} on bench 2; and one local back link, β_1 on bench 1, and β_2 on bench 2 (all photodetectors: 125 MHz bandwidth, 2.5 pW/$\sqrt{\mathrm{Hz}}$ NEP, $\sim$1 μW incident power). The two lasers (Nd:YAG, 500 mW) on each bench were phase locked (unity gain, $f_{\mathrm{ug}} \approx 30$ kHz) with an offset heterodyne frequency (4.25 MHz, bench 1; 3.90 MHz, bench 2). The laser p_{12} was phase locked to p_{21} (as will be done between spacecraft), offset at 4.00 MHz, mimicking a static Doppler shift. White frequency noise was added to the error point of two of the three phase-locking control loops: $\beta_1(t) \approx n_1(t)$ and $\phi_{12}(t) \approx n_2(t)$ at a level of $|n_1(t)|/(2\pi f) = |n_2(t)|/(2\pi f) = 800\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$, mimicking prestabilized noise of that level for the LISA lasers. Because of the short path length, α is first-order insensitive to path length noise in the common optical lengths. Therefore, picometer stability in the arms was not necessary. However, α is sensitive to optical path changes within each ULE bench, e.g., proof-mass displacement.

Several mirrors in the nominally insensitive optical path were dithered at tens of Hz with 1 cycle amplitude triangular waveforms, in order to up convert cyclic nonlinearity from spurious interferometer paths to frequencies above the LISA signal band. The amplitude of the cyclic nonlinearity without dither was approximately 1 nm, and the frequency, equal to the number of wavelengths/second of thermal expansion in the Sagnac path, was in the mHz band. The LISA optical bench will also suffer from cyclic nonlinearity from spurious interference, though of reduced amplitude owing to better control of scatter. Dither on LISA will not be necessary as the intraspacecraft optical paths, by design, should drift by much less than a wavelength.

All data were recorded and processed offline, as will be the case for LISA. For ease of analysis, the phase meters were started with a synchronization pulse. After the start, each clock ran independently. The time evolution of the clocks was measured from the sideband-carrier heterodyne phase, shown in Fig. 3. Trace 3i shows that the clock offset grows to 4 ms over 2500 s (1.6 ppm frequency difference). The detrended clock offset, trace 3ii, shows the relative clock noise.

Figure 4 shows the experimental results, presented as root power spectral densities over the frequency range 0.2 mHz to 3 Hz. Trace 4i shows the signal measured at photodiode β_1; the phase spectrum of the 800 Hz/$\sqrt{\mathrm{Hz}}$ noise injected at the error point of the phase-locking loop derived from β_1. This level of noise is present on all detectors and phase meters, except β_2 which is used as the phase-locking signal for laser p_{32} to p_{12}.

If α is formed without correcting for either clock offset or clock noise, both laser and clock fluctuations couple into the measurement [as in Eq. (2)], shown in trace 4ii. To remove laser frequency noise with unsynchronized clocks, recorded signals from one bench were resampled by interpolation, synchronizing sampling times of the two benches' measurements (we assume negligible arm length, $D_{ij} \rightarrow 1$). Using interpolation algorithms from [11], we interpolated bench 2 signals (the correction is symmetric, either bench could have been chosen):

$$\begin{aligned}\alpha_{\mathrm{int}}(t_1) = {} & D_\Delta(\phi^c_{12}(t_2) - \phi^c_{32}(t_2) - 2\beta^c_2(t_2)) + \phi^c_{31}(t_1)\\ & - \phi^c_{21}(t_1) - 2\beta^c_1(t_1),\end{aligned} \tag{4}$$

Here we have defined the difference of the time measured by the two clocks, $\Delta t(t_1) = t_2 - t_1$ and the delay operator, $D_\Delta a(t) = a(t + \Delta t(t))$. This removes laser frequency noise from α as shown in trace 4iii. However, clock noise is still present and now dominates the spectrum. For spacecraft 1, for example,

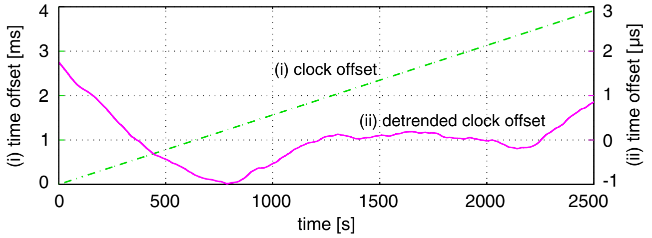

FIG. 3 (color online). Recorded (i) relative clock offset and (ii) detrended clock offset (clock noise), obtained from the sideband-carrier beat note phase measurement.

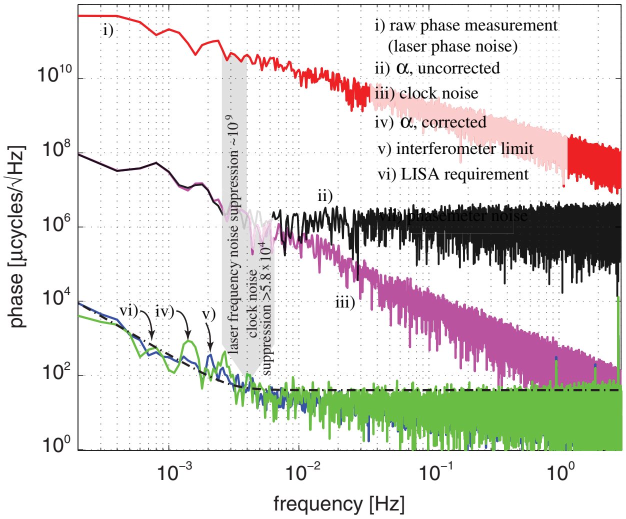

FIG. 4 (color online). Displacement measurements of the LISA interferometry test bed showing (i) injected laser phase noise, (iv) interpolated and clock noise corrected Sagnac TDI variable, demonstrating phase noise cancellation by ~9 orders of magnitude, down to the interferometer noise floor (v). Note: 1 μ cycle ≃ 1 pm displacement equivalent.

$$\phi_{c1}(t_1) = \phi_{sb1}(t_1) - \phi_{21}^{c}(t_1) = (q_2(t_1) - q_1(t_1))f_{12}(t_1), \tag{5}$$

where $f_{12}(t_1)$ is the clock noise sideband microwave frequency. The clock noise and frequency noise free TDI combination is then

$$\alpha(t_1) = \alpha_{\mathrm{int}}(t_1) - \frac{\phi_{c1}(t_1)\nu_{\mathrm{TB}}(t_1)}{f_{12}(t_1)}, \tag{6}$$

where $\nu_{\mathrm{TB}}(t_1) = \nu_{12}(t_1) + \nu_{21}(t_1) - \nu_{31}(t_1) - \nu_{32}(t_1)$. This final TDI output is trace 4iv, reaching a displacement sensitivity similar to the total interferometry budget of LISA, trace 4vi. The final TDI output matches the measured displacement limit of the interferometer, trace 4v; the level with synchronized (phase locked) clocks and no noise injected into the laser phase locking.

The final TDI output shows laser frequency noise was suppressed by approximately 10^9 at 3 mHz, relative to the noise level in the individual phase measurements. LISA requires a laser frequency noise suppression factor of 3×10^7 Hz/f [10]. The clock noise was suppressed by 6×10^4 at 3 mHz, compared to the required suppression factor of 10–1000 (depending on the clock flown).

The original planning documents for LISA [1,15] recognized the problem of suppressing phase noise from lasers and clocks. However, the proposed techniques were impractical and untested. In NASA's 2004 LISA Technology Development Plan, cancellation of laser noise and clock noise were rated 1st and 2nd on the project's 69 item list of interferometry technology risks. It would take the dual breakthroughs of postprocessed TDI and high dynamic range phase meters to realize a design that could be implemented. Work presented here tests these techniques in a configuration similar to LISA, including most of the significant noise effects. The results provide validation of the LISA phase meter, the essential features of the LISA optical design, and evidence that LISA performance will not be limited by technical phase noise.

TDI represents a paradigm shift in the way interferometric measurements are performed. It is a move away from the conventional approach of stabilizing prior to measuring. Instead TDI allows measurements to be made using a noisy light source and relies on common-mode rejection of the noise by postprocessing. By shifting the burden of noise rejection from hardware to signal processing, TDI has the potential to drastically simplify many interferometric measurement systems.

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